Branch : Mechanical engg. Semester – 4th

Sub : 1625405

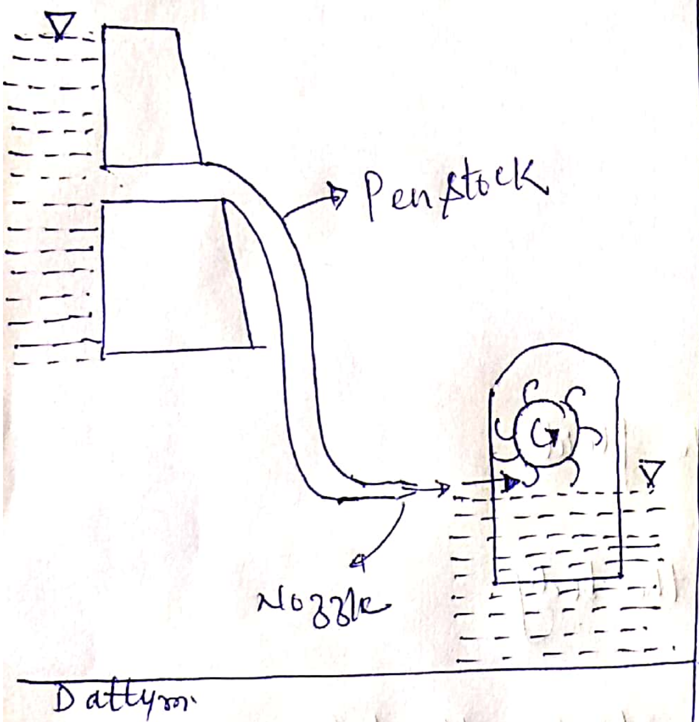
Fluid Mechanics and Machinery

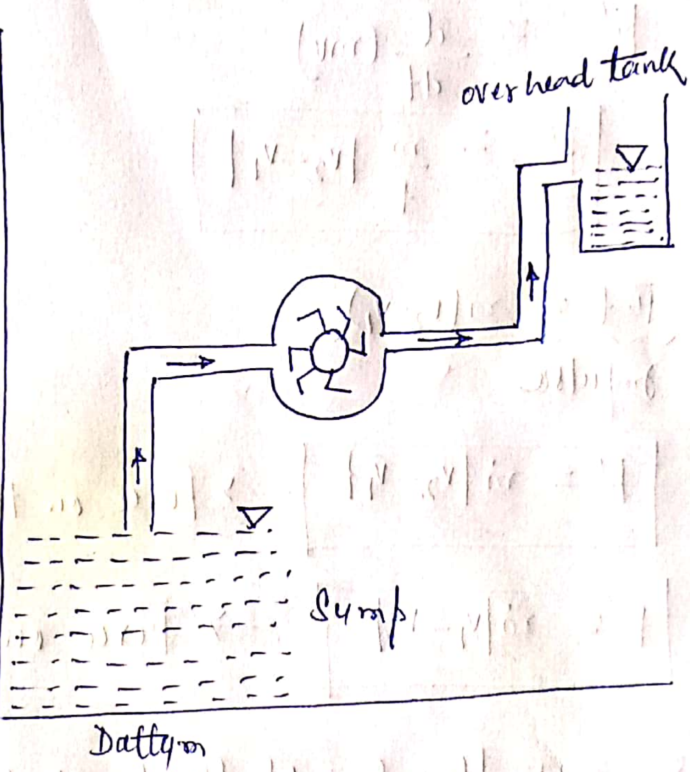
UNIT – 05

Impact of jet

Fluid machinery → Work producing device (turbine)

Fluid machinery → Work consuming device (Pump)





Hydraulic energy → Mechanical energy → electrical energy

Electrical energy → Mechanical energy → Hydraulic energy

Fluid dynamics → It is the study of fluid motion while considering the force causing it.

• The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's 2nd law of motion or from impulse - momentum equation.

$$F = \text{Rate of change of momentum}$$

⇒ momentum = $\underline{\underline{m \cdot V}}$

hydrodynamic force

$$F = \frac{d}{dt}(mv)$$

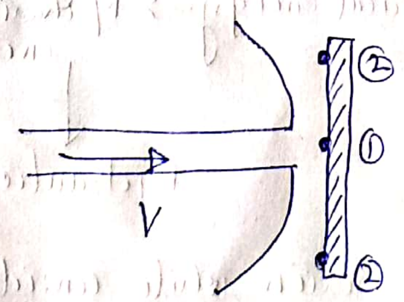
$$\boxed{F = \frac{m}{t}\,|v_2 - v_1|}$$

$$\Rightarrow \underbrace{F \cdot t}_{\text{Impulse}} = m(v_2 - v_1)$$

$$\Rightarrow \boxed{F = \dot{m}\,|v_2 - v_1|} \longrightarrow \text{Force on the fluid}$$

$$\& \quad \boxed{F = \dot{m}\,|v_1 - v_2|} \longrightarrow \text{Force on the body}$$

$\dot{m}$ = mass flow rate of fluid which strikes the body.



① → entry point

② → exit point

Force on the body in + x-direction $\boxed{(F_x) = \dot{m}\,|V_{1x} - V_{2x}|}$

Force on the body in + y direction $\boxed{(F_y) = \dot{m}\,|V_{1y} - V_{2y}|}$

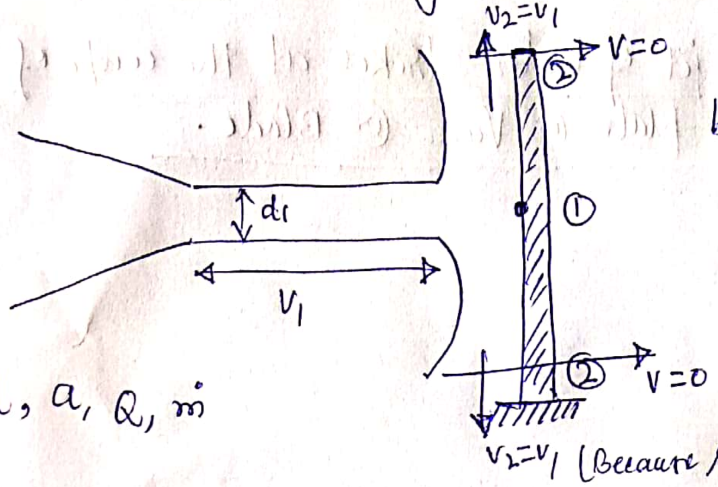
Case-1 Force exerted by the jet on a stationary vertical plate

Assumptions:

① The body is smooth, therefore no change in speed.

② The change in datum is very small, therefore neglected

③ Impact losses are neglected.



$\rho, d, a, Q, \dot{m}$

mass of fluid strikes the body per second $(\dot{m}) = \underline{\rho a V_1}$

$$F_x = \dot{m}\,|V_{1x} - V_{2x}| = \rho a V_1\,|V_1 - 0| = \boxed{\rho a V_1^2} \text{ Newton}$$

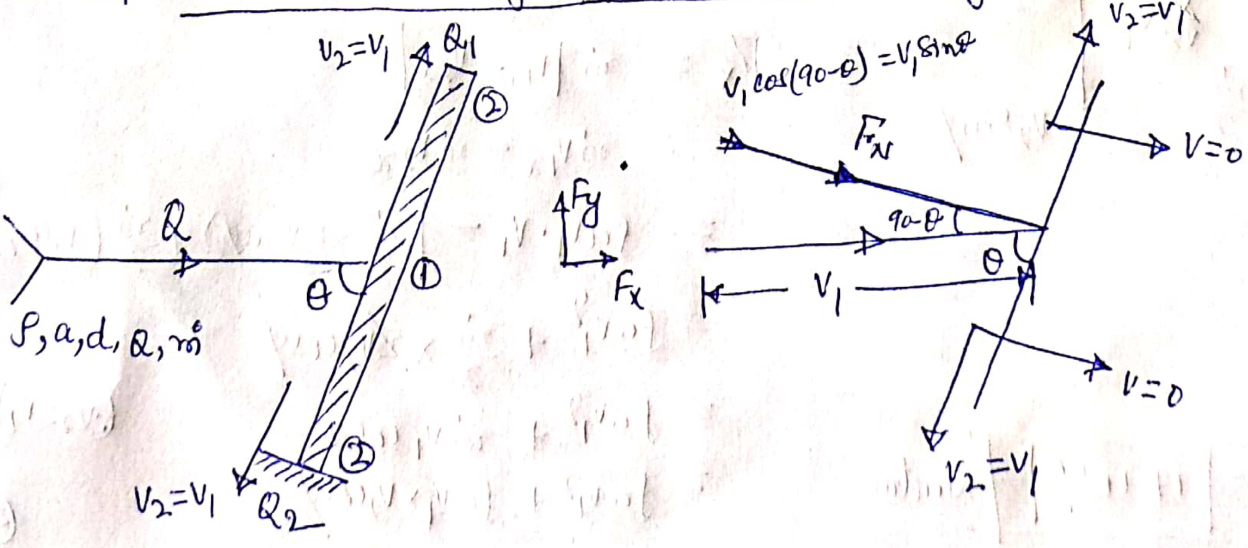
$$F_y = \dot{m}\,|V_{1y} - V_{2y}| = \dot{m} V_{1y} - \dot{m} V_{2y} = (\rho a V_1 \times 0) - \left|\frac{\dot{m}}{2}V_2 + \frac{\dot{m}}{2}(-V_2)\right|$$

$$= 0$$

$$\boxed{F_y = 0}$$

Plate is stationary, so, Work done is zero.

Case-2 Force exerted by a jet on a stationary inclined flat plate



$\dot{m} = \rho a v_1$

$F_N = \rho a v_1 \left| v_1 \sin\theta - 0 \right|$

$\boxed{F_N = \rho a v_1^2 \sin\theta \text{ Newton}}$

$\boxed{\text{Workdone} = 0}$

$F_x = F_N \cos(90^\circ - \theta)$

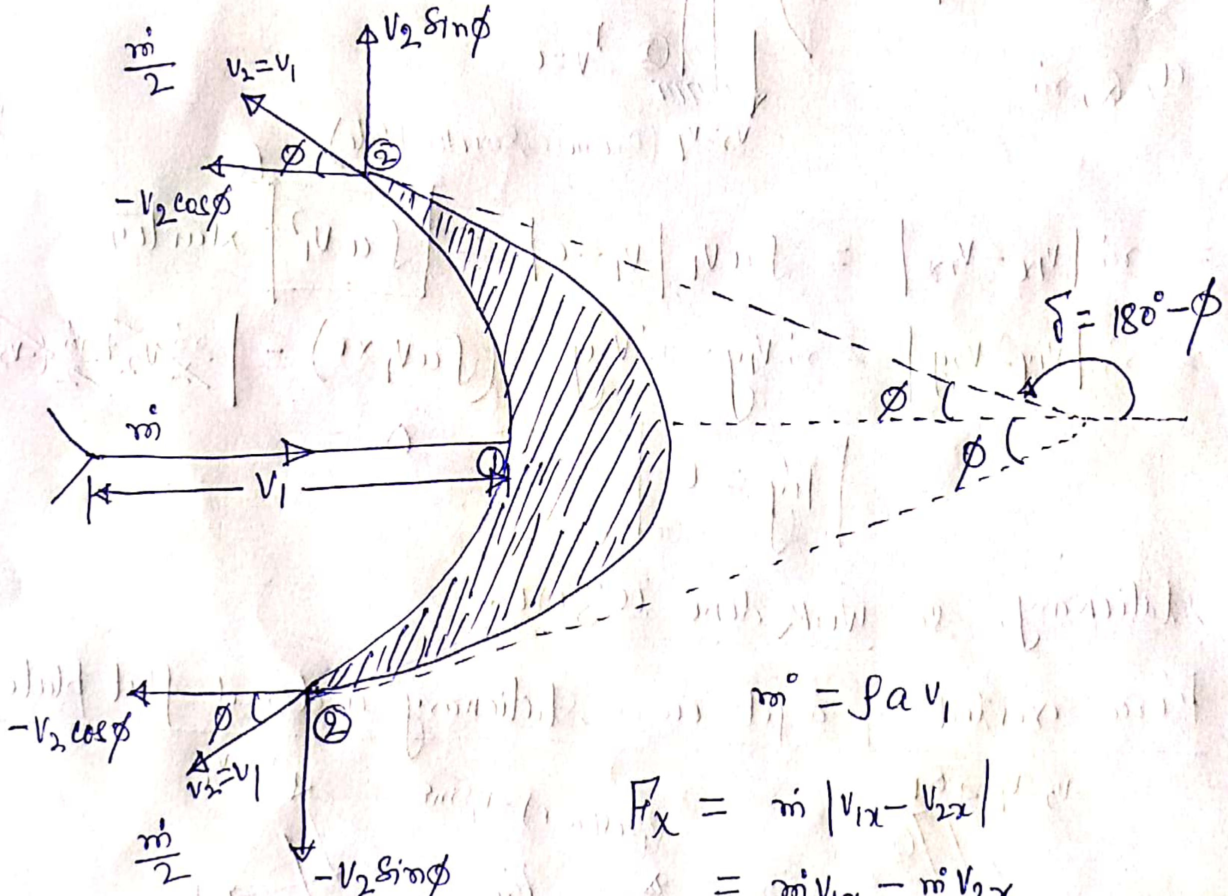
$= \rho a v_1^2 \sin\theta \times \sin\theta$

$= \rho a v_1^2 \sin^2\theta \quad (N)$

$F_y = F_N \sin(90 - \theta)$

$= F_N \cos\theta = \rho a v_1^2 \sin\theta \cos\theta \quad (N)$

Case-3 Force exerted by jet of water strikes at the centre of a stationary curved plate (or) Vane (or) Blade.



$\dot{m} = \rho a v_1$

$F_x = \dot{m} \left| v_{1x} - v_{2x} \right|$

$= \dot{m} v_{1x} - \dot{m} v_{2x}$

$= \rho a v_1 \cdot v_1 - \left| \frac{\dot{m}}{2} \times (-v_2 \cos\phi) + \frac{\dot{m}}{2}(-v_2 \cos\phi) \right|$

$= \rho a v_1^2 + \frac{\dot{m}}{\cancel{2}} \times \cancel{2} v_2 \cos\phi$

$= \rho a v_1^2 + \dot{m} v_1 \cos\phi \quad (\because v_2 = v_1)$

$= \rho a v_1^2 + \rho a v_1 \times v_1 \cos\phi$

$\boxed{F_x = \rho a v_1^2 \left| 1 + \cos\phi \right| \text{ Newton}}$

$$F_y = \dot{m}\,|v_{1y} - v_{2y}| = \underbrace{\rho a v_1 \times v_{1y}}_{} - \dot{m} \times v_{2y}$$

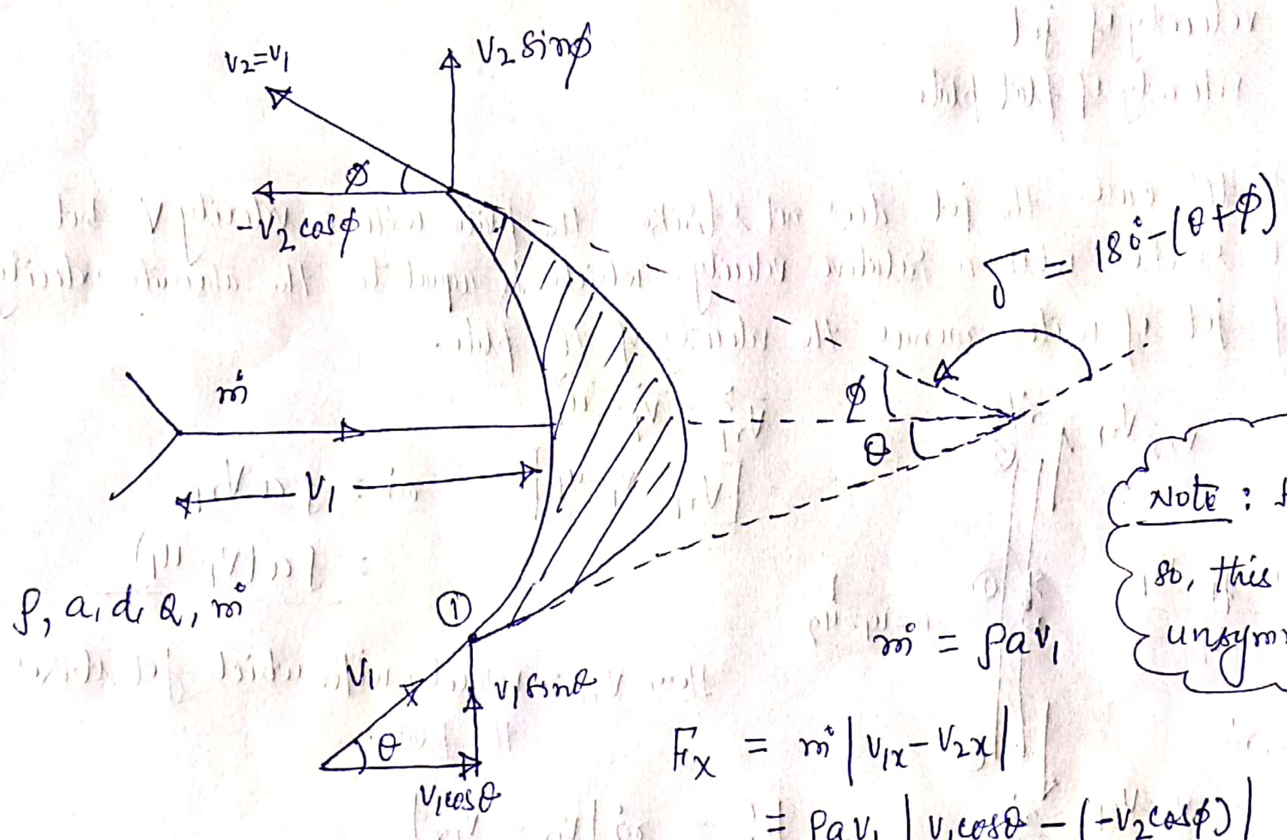
$$= (\rho a v_1 \times 0) - \left| \frac{\dot{m}}{2}\cancel{v_2 \sin\phi} + \frac{\dot{m}}{2}\cancel{(-v_2 \sin\phi)} \right|$$

$$= 0 - 0 = 0$$

$\boxed{F_y = 0}$

$\boxed{\text{Work done} = 0}$ Because plate is stationary.

$\boxed{\text{Case- 4}}$ **Force exerted by jet of water strikes stationary curved plate in tangential direction:**



Note : Here, $\theta \neq \phi$ so, this vane is unsymmetric.

$$\dot{m} = \rho a v_1$$

$$F_x = \dot{m}\,|v_{1x} - v_{2x}|$$

$$= \rho a v_1 \,|v_1 \cos\theta - (-v_2 \cos\phi)|$$

$$F_x = \rho a v_1 \,|v_1 \cos\theta + v_1 \cos\phi| \qquad (\because v_2 = v_1)$$

$$\boxed{F_x = \rho a v_1^2 \,|\cos\theta + \cos\phi|} \text{ Newton}$$

$$F_y = \dot{m}\,|v_{1y} - v_{2y}| = \rho a v_1 \,|v_1 \sin\theta - v_2 \sin\phi|$$

$$= \rho a v_1 \,|v_1 \sin\theta - v_1 \sin\phi| \qquad (\because v_2 = v_1)$$

$$\boxed{F_y = \rho a v_1^2 \,|\sin\theta - \sin\phi|} \text{ Newton}$$

Work done is zero. Because plate is stationary.

- **For symmetric Vane :** $\boxed{\theta = \phi}$

$$\delta = 180^\circ - 2\theta = 180^\circ - 2\phi$$

$$F_x = 2\rho a v_1^2 \cos\theta$$

$$F_y = 0$$

Workdone = zero.

Case-5 Force exerted by jet of water striking moving flat plate :

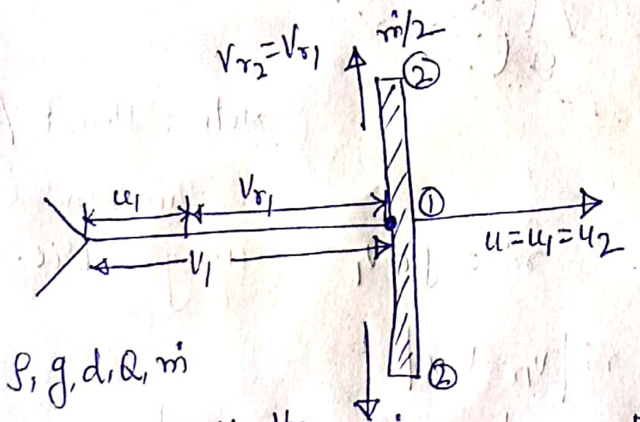
v = velocity of jet
u = velocity of flat plate

Note : In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

$$\vec{V_{r_1}} = \vec{V_1} - \vec{u_1}$$

$$\boxed{V_{r_1} = V_1 - u_1}$$

$$\dot{m} = \rho a V_{r_1} = \rho a (V_1 - u_1)$$



Here V_{r_1} = velocity with which jet strikes the plate.

$\rho, g, d, Q, \dot{m}$

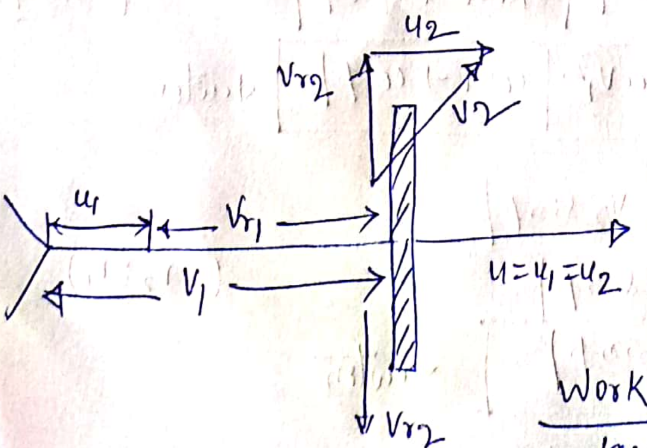
$$F_x = \dot{m}\,|V_{1x} - V_{2x}|$$

$$= \rho a (V_1 - u_1)\,|V_1 - u_2|$$

$$= \rho a (V_1 - u_1)\,|V_1 - u_1| \quad (\because u_1 = u_2 = u)$$

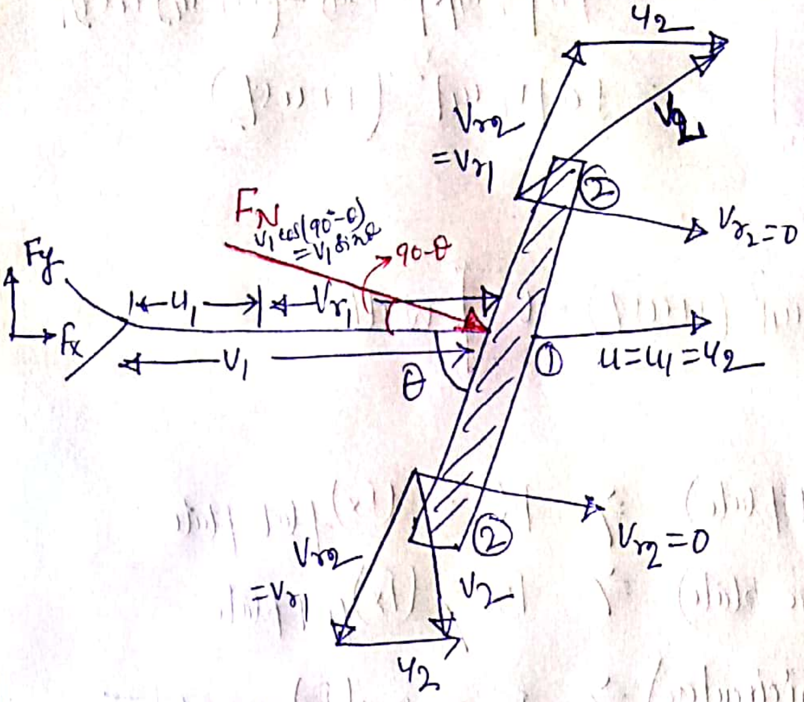
$$\boxed{F_x = \rho a |V_1 - u_1|^2}$$

$$\boxed{F_y = 0}$$



$$\frac{\text{Workdone}}{\text{second}} = F_x \cdot u = \boxed{\rho a |V_1 - u_1|^2 \times u} \ \text{watt or } \frac{N\text{-}m}{s}$$

Case-6 Force exerted by jet of water striking moving inclined plate :



$$\vec{V_{r_1}} = \vec{V_1} - \vec{u_1}$$

$$\boxed{V_{r_1} = V_1 - u_1}$$

$$\dot{m} = \rho a V_{r_1} = \rho a (V_1 - u_1)$$

$$F_N = \dot{m}\,|V_{r_1 N} - V_{r_2 N}|$$

$$= \rho a (V_1 - u_1)\,|V_{r_1}\sin\theta - 0|$$

$$= \rho a (V_1 - u_1)\,|(V_1 - u_1)\sin\theta|$$

$$\boxed{F_N = \rho a |V_1 - u_1|^2 \sin\theta} \quad \text{Newton}$$

$$F_x = F_N \cos(90-\theta)$$

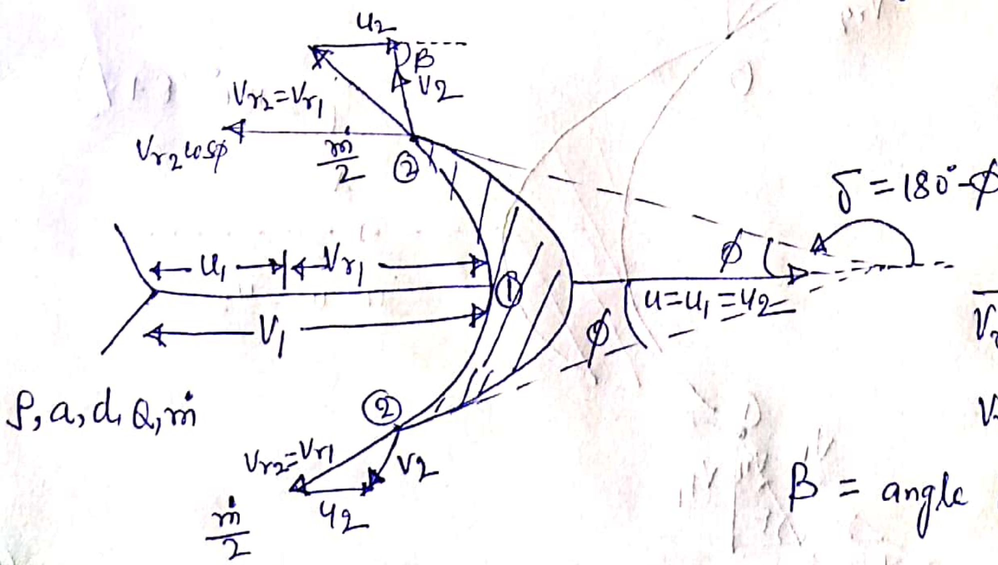
$$= F_N \sin\theta = \boxed{\rho a |V_1 - u_1|^2 \sin^2\theta = F_x}$$

$$F_y = F_N \sin(90^\circ-\theta)$$

$$= F_N \cos\theta = \boxed{\rho a |V_1 - u_1|^2 \sin\theta \cdot \cos\theta = F_y}$$

$$\frac{\text{Workdone}}{\text{second}} = F_x \times u = \boxed{\rho a |V_1 - u_1|^2 \sin^2\theta \cdot u \quad \text{Watt}}$$

Case-7 Force exerted by jet of water striking centre of moving curve plate:



$$\vec{V_{r_1}} = \vec{V_1} - \vec{u_1}$$

$$V_{r_1} = V_1 - u_1$$

β = angle between V_2 and u_2.

$\dot{m} = \rho a V_{r_1} = \rho a |v_1 - u_1|$

$F_x = \dot{m} \left|(V_{r_1})_x - (V_{r_2})_x\right|$

$= \rho a |v_1 - u_1| \times \left|(v_1 - u_1) - \{-(v_1 - u_1)\cos\phi\}\right|$

$= \underline{\rho a |v_1 - u_1|^2 (1 + \cos\phi)}$

$$\boxed{F_x = \rho a |v_1 - u_1|^2 (1 + \cos\phi)}$$ Newton

$$\boxed{F_y = 0}$$

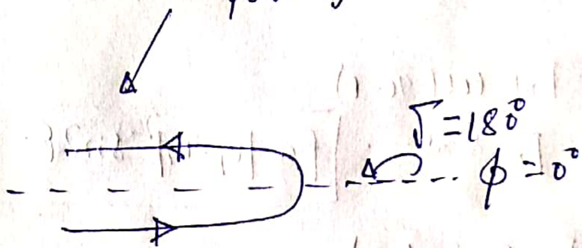
$\dfrac{\text{Work done}}{\text{second}} = F_x \cdot u = \boxed{\rho a |v_1 - u_1|^2 (1 + \cos\phi) \cdot u}$ watt

~~Case-8~~

If $\phi = 90°$ (for flat plate) $\Rightarrow F_x = (F_x)_{\text{flat plate}}$

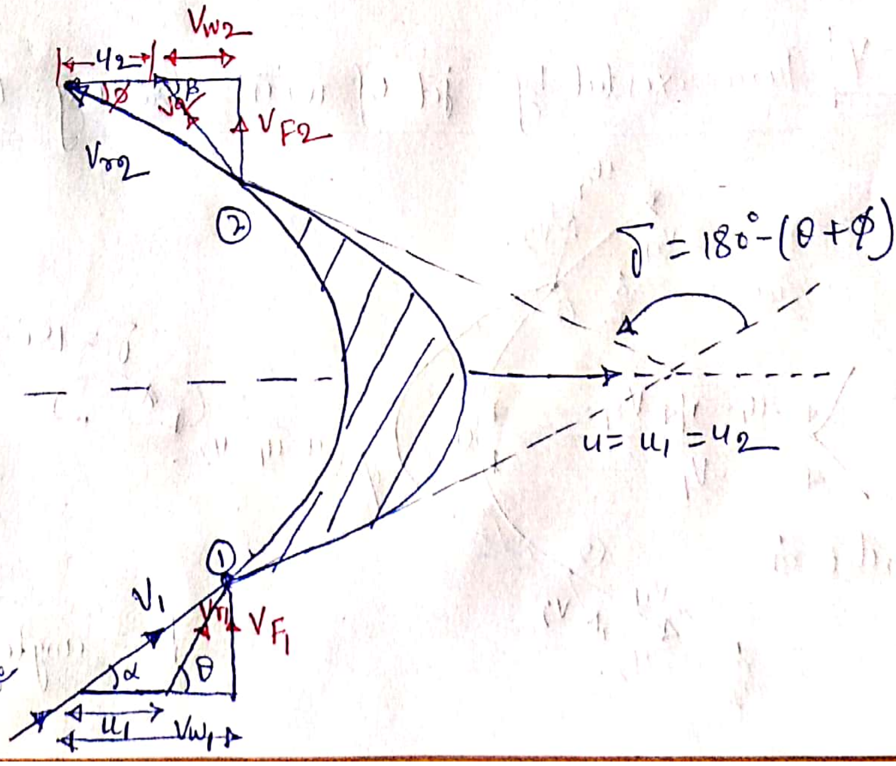
If $\phi < 90°$ (for curve plate) $\Rightarrow F_x > (F_x)_{\text{flat plate}}$

If $\phi = 0°$ (for semicircular plate) $\Rightarrow F_x = 2 \times (F_x)_{\text{flat plate}}$



Case-8 Force exerted by jet of water striking to a moving vane in tangential direction :

Unsymmetrical Vane



- Smooth Vane

$V_{r_2} = V_{r_1}$

- Rough Vane

$V_{r_2} < V_{r_1}$

$V_{r_2} = K \cdot V_{r_1}$ → co-efficient of friction

α = Nozzle angle

θ = Vane angle at entry

ϕ = Vane angle at exit

β = angle between V_2 and u_2.

V_1, V_2 = absolute velocity of jet at entry and exit

V_{r_1}, V_{r_2} = relative velocity of jet at entry and exit

u_1, u_2 = Vane velocity at entry and exit

V_{w_1}, V_{w_2} = tangential component of absolute velocity at entry and exit

V_{F_1}, V_{F_2} = Radial component of absolute velocity at entry and exit.

$$\dot{m} = \rho a \vec{V}_{r_1} = \rho a |\vec{v}_1 - \vec{u}_1|$$

$$F_x = \dot{m}|(V_{r_1})_x - (V_{r_2})_x| = \rho a V_{r_1} |V_{r_1}\cos\theta - (-V_{r_2}\cos\phi)|$$

$$= \rho a V_{r_1} |\underline{V_{r_1}\cos\theta} + \underline{V_{r_2}\cos\phi}|$$

$$= \rho a V_{r_1} |V_{w_1} \cancel{- u_1} + V_{w_2} \cancel{+ u_2}| \qquad (\because u_1 = u_2 = u)$$

$$= \rho a V_{r_1} |V_{w_1} + V_{w_2}|$$

$$\boxed{F_x = \rho a V_{r_1} |V_{w_1} + V_{w_2}|} \text{ Newton}$$

$$F_y = \rho a V_{r_1} |(V_{r_1})_y - (V_{r_2})_y| = \rho a V_{r_1} |V_{r_1}\sin\theta - V_{r_2}\sin\phi|$$

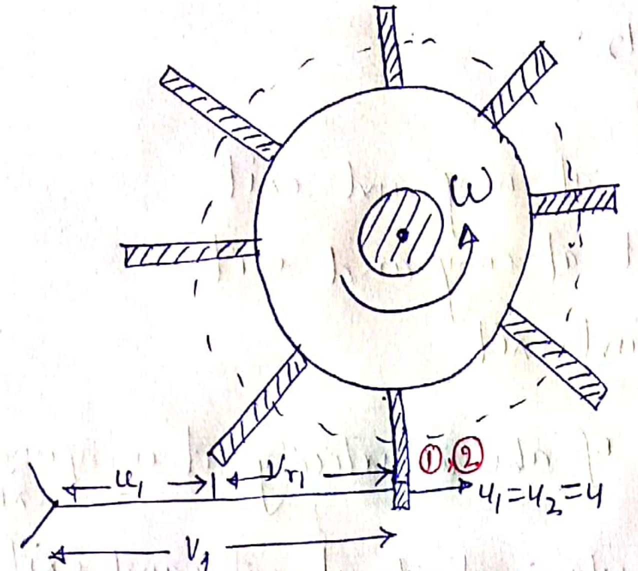
$$\boxed{F_y = \rho a V_{r_1} |V_{F_1} - V_{F_2}|} \text{ Newton}$$

$$\frac{\text{Work done}}{\text{second}} = F_x \cdot u = \underline{\underline{\rho a V_{r_1} |V_{w_1} + V_{w_2}| \cdot u}} \text{ watt}$$

- Idea flow : for ideal flow, $V_{F_1} = V_{F_2}$ so, $\boxed{F_y = 0}$ Always required Because radial thrust not required.
- Symmetrical vane : $\theta = \phi$, $\underline{\gamma = 180° - 2\theta} = \underline{180° - 2\phi}$

Scanned by CamScanner

Case - 9 Force exerted by jet of water striking flat plate mounted on a wheel.



D = Pitch circle diameter of the wheel.

N = speed (RPM)

$u = \frac{\pi D N}{60} = u_1 = u_2$

$\boxed{\dot{m} = \rho a V_1}$ ***

~~Not~~ $\dot{m} \neq \rho a V_{r_1}$

In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered. Hence mass of water per second striking the series of vanes $= \rho a V_1$.

$\rho, a, d, Q, \dot{m}$

$a = \frac{\pi}{4} d^2$

$$F_x = \dot{m}\,|(V_{r_1})_x - (V_{r_2})_x|$$

$$= \rho a V_1\,|V_{r_1} - 0| = \rho a V_1\,|V_1 - u_1|$$

$\boxed{F_x = \rho a V_1 |V_1 - u_1|}$ Newton * , $\boxed{F_y = 0}$ * (so, no axial thrust)

$$\left(\frac{\text{Work done}}{\text{second}}\right) = F_x \cdot u = \rho a V_1 |V_1 - u_1| \cdot u \text{ watt}$$

- $$\text{efficiency}(\eta) = \frac{\text{WD/second}}{(\text{Kinetic energy at entry})/\text{second}} = \frac{\rho a V_1 |V_1 - u_1| \cdot u}{\frac{1}{2} \cancel{\rho a} V_1^2} = \frac{2|V_1 - u_1| \cdot u}{V_1^2}$$

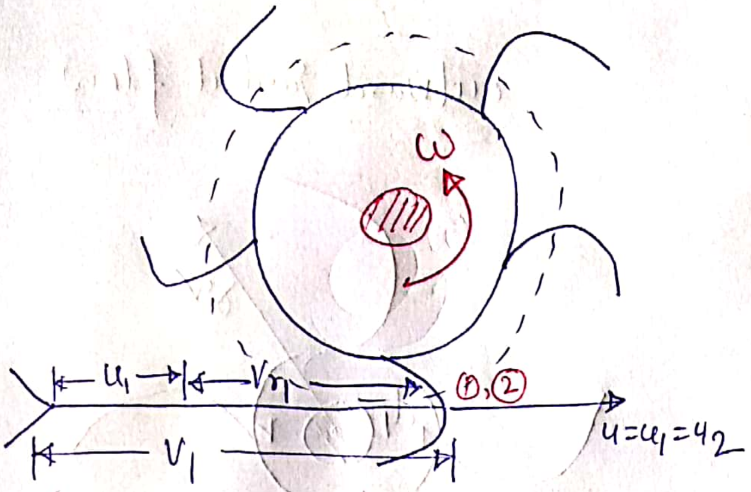
- Now for a given V_1

$$\frac{d\eta}{du} = 0 \Rightarrow 0 = \frac{2|V_1 - u_1|}{V_1^2} \Rightarrow u = \frac{V_1}{2}$$

* → efficiency will be max^m when $u = \frac{V_1}{2}$.

So, $$\eta_{max} = \frac{2\left|V_1 - \frac{V_1}{2}\right| \frac{V_1}{2}}{V_1^2} \Rightarrow \boxed{\eta_{max} = \frac{1}{2} = 50\%}$$ *

Case-10 Force exerted by jet of water striking to the centre of curved vane mounted on a wheel:



D = pitch circle diameter of wheel

N = speed (RPM)

$u = \frac{\pi D N}{60} = u_1 = u_2$

$$\boxed{\dot{m} = \rho a V_1} \;**$$

$\rho, g, d, a, Q, \dot{m}$

$$\boxed{F_x = \rho a V_1 |V_1 - u_1| |1+\cos\phi|} \text{ Newton}$$

$\boxed{F_y = 0}$ No axial thrust

- $\frac{\text{Work done}}{\text{second}} = F_x \times u = \rho a V_1 |V_1 - u_1| |1+\cos\phi| \cdot u$ watt

- $\eta = \frac{\text{WD/second}}{(\text{K.E. of fluid at entry per second})} = \frac{\rho a V_1 |V_1 - u_1| |1+\cos\phi| \cdot u}{\frac{1}{2} \dot{m} V_1^2}$

$$\boxed{\eta = \frac{2|V_1 - u_1| |1+\cos\phi| u}{V_1^2}}$$

- for a given V_1,

$\frac{d\eta}{du} = 0 \Rightarrow \boxed{u = \frac{V_1}{2}}$ * $\boxed{\eta_{max} = \frac{1+\cos\phi}{2}}$ **

- If $\phi = 90°$ (for flat plate). $\eta_{max} = \frac{1}{2} = 50\%$
- If $\phi < 90°$ (curved plate), $\eta_{max} = 99.2\%$

let $\phi = 10°$

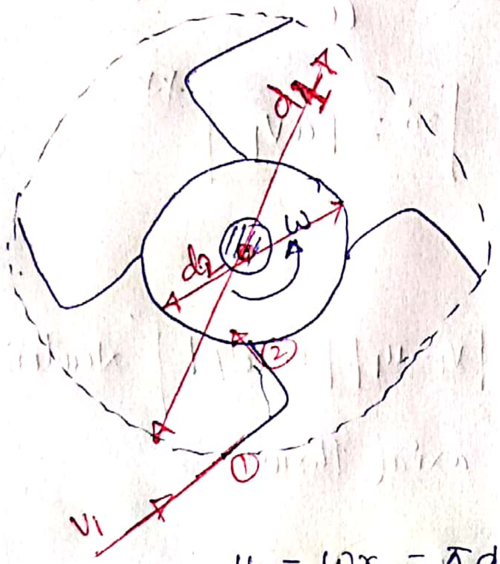
- If $\phi = 0$, (semi circular plate), $\eta_{max} = 100\%$

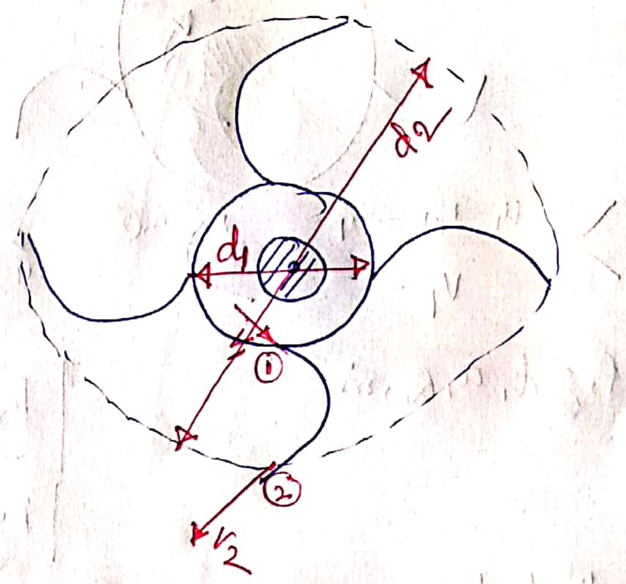
Case - II

Jet of water striking the vane in tangential direction mounted on a wheel.

Inward radial flow

outward radial flow





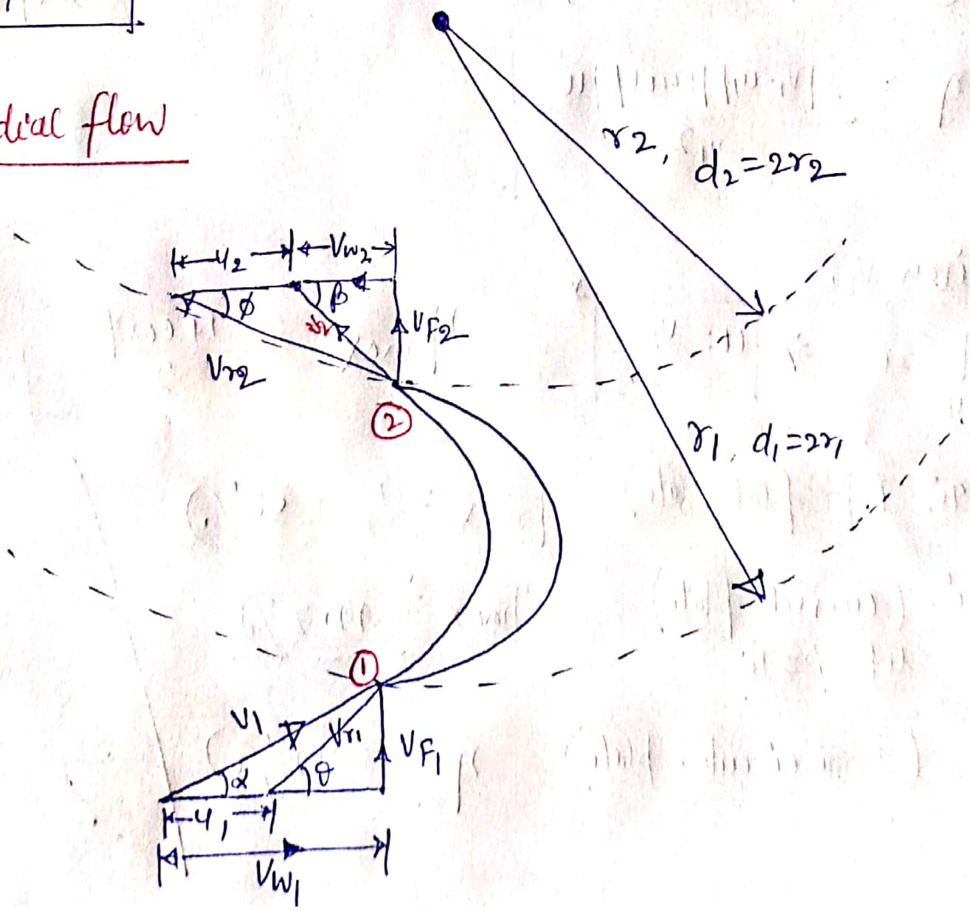
$$u_1 = \omega r_1 = \frac{\pi d_1 N}{60}$$

$$u_2 = \omega r_2 = \frac{\pi d_2 N}{60}$$

$d_1 \neq d_2$

$\therefore \boxed{u_1 \neq u_2}$ **

Inward radial flow



$u_1 = \omega r_1 = \frac{\pi d_1 N}{60}$

$u_2 = \omega r_2 = \frac{\pi d_2 N}{60}$

$d_1 \neq d_2$

So, $\boxed{u_1 \neq u_2}$

- Smooth Vane.

$V_{r_2} = V_{r_1}$

- Rough Vane

$V_{r_2} < V_{r_1}$

$V_{r_2} = K \cdot V_{r_1}$

$\dot{m} = \rho a V_1$

Rotor power = $T \cdot \omega$

Torque $\longrightarrow$ Rate of ~~&~~ change of angular momentum

Angular momentum = Moment of linear momentum

→ Linear momentum per second of fluid at entry in tangential direction.

$= \dot{m} V_{w_1}$

So, angular momentum per second at entry $= \dot{m} V_{w_1} \cdot r_1$

→ Linear momentum per second at exit in tangential direction $= -\dot{m} V_{w_2}$

So, angular momentum per second at exit $= -\dot{m} V_{w_2} r_2$

~~&~~ Hence, Torque $(T) = \dot{m} V_{w_1} r_1 - (-\dot{m} V_{w_2} r_2)$

$$\boxed{T = \dot{m} \left| V_{w_1} r_1 + V_{w_2} r_2 \right|} \quad N\text{-}m$$

Rotor power $= T \cdot \omega = \rho a v_1 \left| V_{w_1} r_1 + V_{w_2} r_2 \right| \cdot \omega$

$a v_1 = Q$

$\omega r_1 = u_1$

$\omega r_2 = u_2$

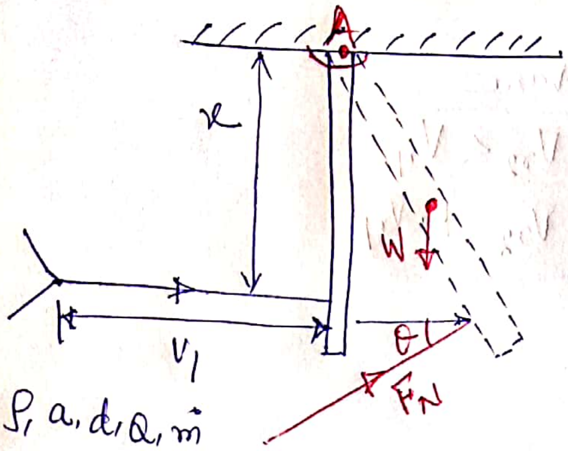
$\boxed{R.P. = \rho Q \left| V_{w_1} u_1 + V_{w_2} u_2 \right| \text{ watt}}$ when, V_{w_2} and u_2 are in opposite direction.

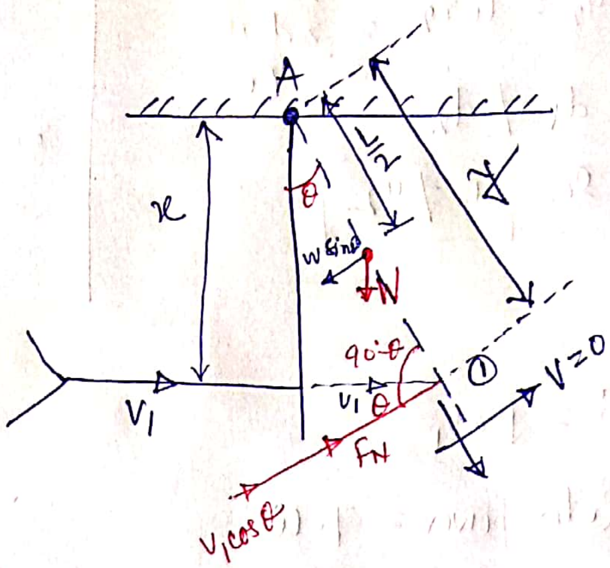
$\boxed{R.P. = \rho Q \left| V_{w_1} u_1 - V_{w_2} u_2 \right| \text{ watt}}$ when, V_{w_2} and u_2 are in same direction.

Note: Work done per unit weight of water entering $= \frac{1}{2}(V_{w_1} + V_{w_2}) \cdot u_1$

$\therefore \boxed{R.P. = \rho Q \left| V_{w_1} u_1 \pm V_{w_2} u_2 \right| \text{ watt}}$ **

Case-12 Jet of water striking vertical hanging plate :





W = Weight of plate

L = Length of plate

At equilibrium condition $\boxed{\sum M_A = 0}$ Sum of moments of all the forces about hinged point A is zero

$$F_N \cdot y = W\sin\theta \cdot \frac{L}{2} \quad \text{——— ①}$$

$$\cos\theta = \frac{x}{y} \quad \Rightarrow y = \frac{x}{\cos\theta}$$

and $F_N = \rho a v_1 |v_1\cos\theta - 0| = \rho a v_1^2 \cos\theta$, $\dot{m} = \rho a v_1$

from eqn ①. Putting values of y and F_N,

$$\rho a v_1^2 \cancel{\cos\theta} \cdot \frac{x}{\cancel{\cos\theta}} = W\sin\theta \times \frac{L}{2}$$

$$\sin\theta = \frac{2\rho a v_1^2 x}{WL}$$

$$\boxed{\theta = \sin^{-1}\left[\frac{2\rho a v_1^2 x}{WL}\right]}$$

special case, when $x = \frac{L}{2}$

then $\boxed{\theta = \sin^{-1}\left|\frac{\rho a v_1^2}{W}\right|}$